Cantilever Beam Scale

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Abstract — A strain-gauge cantilever scale was built and evaluated against four labeled reference loads. The objectives were to compare mechanics-based and calibrated weight estimates, quantify placement repeatability, and measure the water removed in four gulps. The course-specified gauge-factor control of 2.085 was used throughout. Calibration gave a slope of −9.76 ± 0.45 N/V. Ten full-bottle placements gave mean weights of 2.50 ± 0.16 N by calibration and 2.50 ± 0.11 N by mechanics, against a balance reference of 2.4759 ± 0.0005 N, differences of 1.1% and 0.9%. The two methods were not distinguishable in accuracy, and the placement standard deviation was 0.017 N. Four gulps measured 0.45 to 0.81 N, each with an uncertainty near 0.17 N, and the calibrated empty-bottle reading, 0.08 ± 0.16 N, was smaller than its uncertainty. A linear beam model gave a first-yield estimate of 42.018 ± 0.070 N from dimensional and course-rule material bounds, whereas the amplifier window limited use to 6.30 ± 0.29 N, a slope-only extrapolation. Uncertainties are 95% intervals dominated by the tared voltage and calibration slope. As a sensitivity case, the 2.0 ± 1% package gauge factor would raise the mechanics weights by 4.25%.

Index Terms — Calibration, cantilever beam, strain gauge, uncertainty.

I. Introduction

This lab built a weighing scale from a strain-gauge cantilever and used it two ways: a mechanics-based estimate from the beam equations, and a calibrated estimate from a line fitted to known weights. The objectives were to compare the two estimates, quantify repeatability over ten placements of a water bottle, measure the water removed by four gulps [1], and estimate the load limits of the beam. Root-sum-square (RSS) propagation combines independent input uncertainties through a measurement equation. The least-squares calibration slope has no simple closed-form uncertainty equation, so Monte Carlo simulation, which disturbs the inputs repeatedly and refits the line each time, is used to estimate the uncertainty of the slope and intercept [13].

The course identifies the beam as 6061-T6 aluminum [3], [4]. For small elastic bending of a rectangular section, surface stress σ and longitudinal strain ε are related to weight W, gauge-to-load distance L, width b, thickness h, and modulus E by (1), with geometry defined in Fig. 1. Strain is fractional length change. A fixed support and transverse loading are assumed.

$$\sigma = E\varepsilon = \frac{6WL}{bh^2} \tag{1}$$

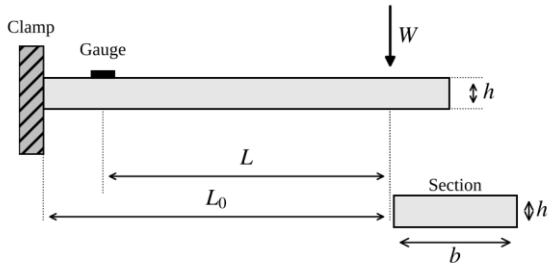


Fig. 1. Cantilever geometry: weight W at distance L from the gauge center and L_0 from the clamp, thickness h, and width b. These distances and section dimensions are the inputs to (1) and (7); thickness is exaggerated.

A bonded gauge changes resistance according to (2), where R_0 is unloaded resistance, ΔR is its change, and G_f is gauge factor. One active gauge and three completion resistors form two voltage dividers (Fig. 2). Their midpoint imbalance produces bridge voltage V_g [3], [4]. Potentiometer trimming brings the unloaded output near zero so amplification does not consume the input range before loading. At gain A of 900, a 0.7 mV bridge offset would occupy approximately the entire ±0.64 V amplifier window. Amplification converts a full-bottle bridge change near 0.29 mV into about 0.26 V, so the signal spans about 3,300 increments of the analog-to-digital converter (ADC).

$$\Delta R = R_0 G_f \varepsilon \tag{2}$$

Equation (3) defines strain from amplified voltage V_a, excitation V_s, and tared voltage ΔV_a. The fixed unloaded offset is V_0. Taring removes the initial electrical offset and the beam's own weight, so ΔVa responds only to the added load; it does not remove subsequent drift.

$$\varepsilon = \frac{4\Delta V_a}{A G_f V_s};\ \Delta V_a = V_a - V_0 \tag{3}$$

Downward loads gave negative electrical readings, so (4) is written to report weight as a positive number. It defines the mechanics weight W_m. Calibrated weight W_c uses slope C with the same tare in (5), as specified by the procedure [1, p. 14]. The fitted intercept B is assessed with the calibration line but is not added to the tared weighing equation.

$$W_m = -\frac{Ebh^2}{6L}\varepsilon \tag{4}$$

$$W_c = C(V_a - V_0) \tag{5}$$

Digital data acquisition samples voltage at discrete times and converts it to integer codes through the ADC. Equation (6) gives increment q from bit depth n and window limits V_{min} and V_{max}. Narrow windows improve resolution but must accommodate the signal without clipping (saturation).

$$q = \frac{V_{max} - V_{min}}{2^n} \tag{6}$$

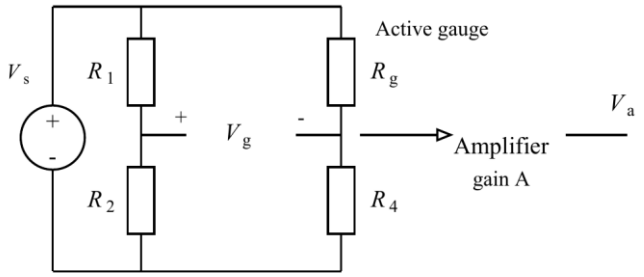


Fig. 2. Simplified quarter bridge and amplifier signal path, redrawn from the operating principle in [3], [4]. R_g is the active gauge, so only it changes with strain and the amplified voltage V_a follows beam bending. Trim circuitry is omitted; lead and amplifier polarity determine output sign.

II. Procedure

The gauge location was marked on the prepared beam surface, and the gauge was taped and glued near the fixed end [5]. Lead wires were soldered to a two-conductor cable with a small strain-relief hump. Before anything was powered, the gauge resistance was checked with a Fluke 115 multimeter. The beam was then clamped in the cantilever fixture, and the gauge, quarter-bridge amplifier board, and SADI 2.7 acquisition board were wired as the course procedure showed [1]. A micrometer measured the width and thickness. A NEIKO stainless-steel caliper measured the distance from the gauge center to a scratch mark at the load center, and the distance from the clamp to that mark. Each weight and the inverted bottle were centered on the mark by hand (Fig. 3). With nothing loaded, V_g was trimmed near zero with the bridge-balancing potentiometer, and V_a was trimmed near zero with the amplifier-centering potentiometer [1]. The amplifier voltage was then tared so that each reading reflected only the added load.

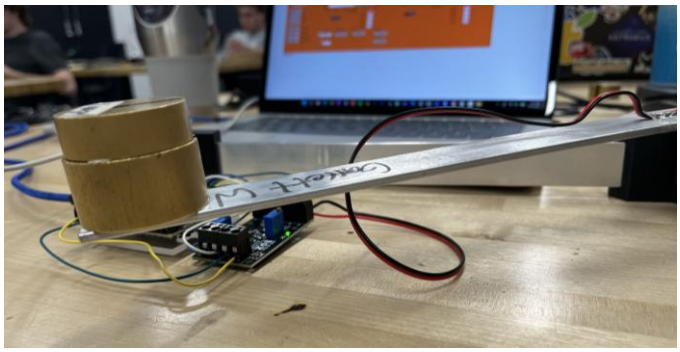


Fig. 3. Cantilever loaded with the 3.0051 N reference weight at the load mark, a distance L from the gauge. Centering every load on the same mark keeps L in (1) fixed; the amplifier and acquisition boards are below the beam.

The LabVIEW virtual instrument (VI) calculated strain and mechanics weight from voltage, geometry, modulus, gain, and the gauge-factor control using (3) and (4) (Fig. 4). Each saved row contained the mean and standard deviation of V_s and V_a, ΔV_a, strain, mechanics weight, and calibrated weight. The control remained 2.085 throughout. Excitation V_s used the ±2.56 V window, the smallest that contained its 2.4991 V mean. V_g, V_a, and the CH3 channel all used the ±0.64 V window, chosen by trial and error as the smallest that did not saturate under the largest calibration load. Sampling at 50,000 samples/s produced means and standard deviations for 2,500-sample, 50 ms blocks. Each calibration file contained three unloaded and three loaded blocks.

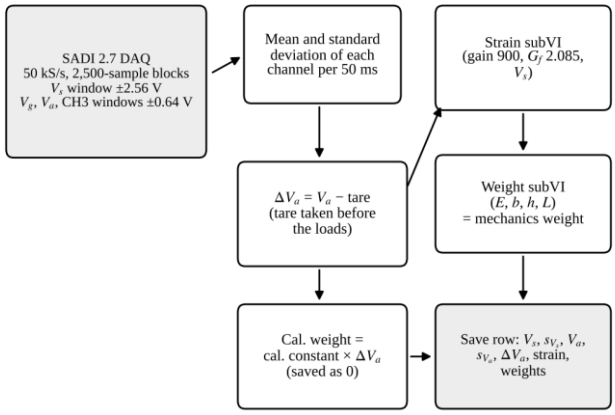


Fig. 4. Weighing VI, redrawn as a block diagram from the LabVIEW back end. Gain 900 and the gauge-factor control enter the strain subVI, so the saved strain and mechanics weight depend on both.

The reference forces were the totals recorded on the tape labels of the brass weights, which had been measured before the session, and no new balance readings were taken. The 3.9286 N load was two 1.9643 N weights stacked together; the other three loads were single weights. Stacked loads were not weighed whole on the balance as the procedure describes [1, p. 14], so their label totals carry the combined uncertainty of their pieces (Appendix). The three loaded voltage means for each force were averaged so every load carried equal weight in the least-squares fit. Known weights were also placed on the beam to check the mechanics-based weight against the 2% to 5% criterion in the procedure [1, p. 14]; the results are given in Results. The saved calibrated-weight columns were all zero, so calibrated weights were computed afterward from the fitted slope and the recorded voltages.

The test object was a small (approximately 8 fl oz) Zephyrhills bottle with its cap on. It was weighed full and empty on a Goldenwall electronic balance that reads to 0.0001 N. Each file began with three unloaded blocks. The placement file then recorded ten full-bottle placements, and the gulp file recorded the full bottle and then the bottle after each of four gulps, with the cap replaced every time. Differences between successive states gave the gulp weights. Uncertainty methods followed the course uncertainty guidance [6], [7].

III. Results

Measured dimensions and acquisition settings are summarized with their assigned uncertainties (Table I). The balance indicated 2.4759 ± 0.0005 N full and 0.0823 ± 0.0005 N empty. The ±0.0005 N bound is the course-specified scale uncertainty [8]. The gauge-factor control was set to 2.085 as the course set-up specified [3]; the gauge packaging listed 2.0 ± 1%, and Section IV treats that difference as a sensitivity case. Before calibration, a multimeter read 120.5 ± 1.3 Ω across the gauge, consistent with the 119.7 ± 0.1 Ω package value [9].

TABLE I
GEOMETRY AND ACQUISITION INPUTS

Quantity	Value / uncertainty
Thickness	3.1370 ± 0.0025 mm
Width	25.3830 ± 0.0025 mm
Gauge-to-load distance	251.390 ± 0.025 mm
Clamp-to-load distance	273.460 ± 0.025 mm
Entered modulus	69.00 ± 0.10 GPa
Amplifier gain	900 ± 4.5 V/V
Control gauge factor	2.085 ± 0.021
Recorded tare voltage	0.006 ± 0.011 V*
ADC bit depth	14 bits
Excitation / amplifier windows	±2.56 V / ±0.64 V
Caliper / micrometer increments	0.01 mm / 0.001 mm

The asterisk identifies a proxy explained in the Appendix. Calculations used the unrounded recorded tare (Appendix).

Calibration gave $C = -9.76 \pm 0.45$ N/V, $B = 0.04 \pm 0.12$ N, and $R^2 = 0.9998$ (Table II; Fig. 5). Calculated ADC increments were 0.313 mV for V_s and 0.0781 mV for V_g, V_a, and CH3, exact for the 14-bit windows; 0.0781 mV corresponds to about 0.0008 N. Saved means were within the windows: the largest $|V_a|$ was 0.40 V against the 0.64 V limit, and V_s had 0.061 V of headroom, so clipping is implausible.

TABLE II
CALIBRATION DATA; THREE LOADED BLOCKS PER FORCE

Load (N)	Mean amplifier voltage (V)	*Largest standard deviation (V)*
1.0479	−0.1016	0.0043
1.9643	−0.1979	0.0059
3.0051	−0.3055	0.0061
3.9286	−0.3962	0.0055

The reference loads are label totals, and their uncertainties are given in the Appendix. For each load the largest within-block standard deviation was used instead of an average of the three, following [6, p. 10], and the voltage uncertainty was twice that value.

The saved mechanics weights of the loaded blocks averaged 1.04 ± 0.14, 1.98 ± 0.15, 3.03 ± 0.15, and 3.92 ± 0.16 N in magnitude. They differ from the labeled loads by −0.4%, +0.9%, +0.9%, and −0.3%, inside the 2% to 5% criterion [1, p. 14] and below the uncertainties of these single-block readings, so the geometry and modulus inputs were not adjusted. The four reference loads had fit residuals of 0.013, −0.010, −0.021, and 0.018 N, each smaller than the roughly 0.12 N coordinate uncertainty.

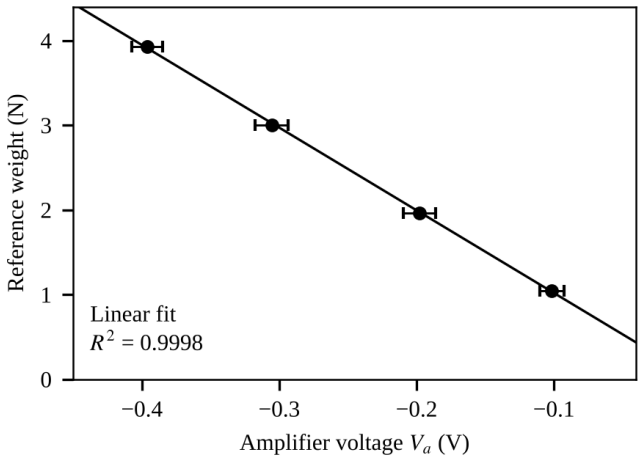


Fig. 5. Reference force versus mean amplifier voltage. The fitted slope converts voltage changes to weight, and the small scatter about the line shows a linear response from 1.05 to 3.93 N. Horizontal bars show twice the largest block standard deviation; the reference-load bounds are smaller than the markers.

The unloaded block means changed from file to file (Table III), which shows the baseline moved during the session. They do not identify which block was the original tare, so the largest within-block standard deviation among them, 0.0056 V, was used as a conservative tare proxy.

TABLE III
UNLOADED AMPLIFIER RECORDS

File (load)	Three block means (mV)	Largest standard deviation (mV)
1.0479 N	7.3, 8.2, 8.4	4.2
1.9643 N	6.9, 6.6, 6.2	4.2
3.0051 N	7.1, 7.5, 6.3	4.2
3.9286 N	8.4, 7.8, 8.7	4.2
Placements	8.7, 7.9, 7.6	5.6
Gulps	5.1, 6.0, 5.6	4.9

Ten placements gave means of 2.50 ± 0.16 N calibrated and 2.50 ± 0.11 N mechanics, including common-input uncertainty. Both sample standard deviations were 0.017 N and repeatability intervals for the means were ±0.013 N. Individual results carry their propagated uncertainty (Table IV; Fig. 6a). Mechanics values reproduce the magnitudes of the saved VI weights; the sign change only makes downward weight positive.

Placement results were stable across the ten placements. Calibrated and mechanics weights both ranged from 2.47 to 2.52 N, and every placement interval contains the balance reference of 2.4759 N (Fig. 6a). The calibrated and mechanics means agree because both rely on the same tared voltage, so their difference reflects only the slope term. The unloaded block means in Table III ranged from 5.1 to 8.7 mV, a spread of 3.6 mV, or about 0.035 N at the calibrated slope. The 0.011 V tare proxy is about three times that spread, which makes it a conservative bound on the baseline movement

TABLE IV
TEN PLACEMENTS; INDIVIDUAL UNCERTAINTIES IN NEWTONS

No.	*Voltage / standard deviation (V)*	*Calibrated weight (N)*	*Mechanics weight (N)*
1	−0.2481 / 0.0044	2.48 ± 0.18	2.47 ± 0.14
2	−0.2518 / 0.0044	2.51 ± 0.18	2.51 ± 0.14
3	−0.2477 / 0.0064	2.47 ± 0.20	2.47 ± 0.17
4	−0.2519 / 0.0062	2.51 ± 0.20	2.51 ± 0.17
5	−0.2504 / 0.0073	2.50 ± 0.21	2.49 ± 0.18
6	−0.2500 / 0.0061	2.49 ± 0.20	2.49 ± 0.16
7	−0.2527 / 0.0060	2.52 ± 0.20	2.52 ± 0.16
8	−0.2516 / 0.0058	2.51 ± 0.20	2.51 ± 0.16
9	−0.2529 / 0.0059	2.52 ± 0.20	2.52 ± 0.16
10	−0.2514 / 0.0056	2.51 ± 0.19	2.50 ± 0.16

The separate gulp sequence decreased from 2.55 ± 0.20 N to 0.08 ± 0.16 N by calibration (Table V; Fig. 6b). Four gulps were 0.61 ± 0.17, 0.59 ± 0.17, 0.45 ± 0.17, and 0.81 ± 0.17 N. Their observed mean was 0.618 ± 0.051 N from propagated measurement uncertainty; sample standard deviation was 0.15 N. A Student-*t* interval of ±0.23 N would describe a different quantity, an assumed population mean estimated from four independent gulps.

TABLE V
BOTTLE SEQUENCE; WEIGHTS AND UNCERTAINTIES IN NEWTONS

State	*Voltage / standard deviation (V)*	*Calibrated weight (N)*	*Mechanics weight (N)*
Full	−0.2558 / 0.0061	2.55 ± 0.20	2.55 ± 0.16
After 1	−0.1929 / 0.0061	1.94 ± 0.18	1.93 ± 0.16
After 2	−0.1320 / 0.0064	1.34 ± 0.18	1.34 ± 0.17
After 3	−0.0855 / 0.0060	0.89 ± 0.17	0.89 ± 0.16
Empty	−0.0027 / 0.0062	0.08 ± 0.16	0.08 ± 0.16

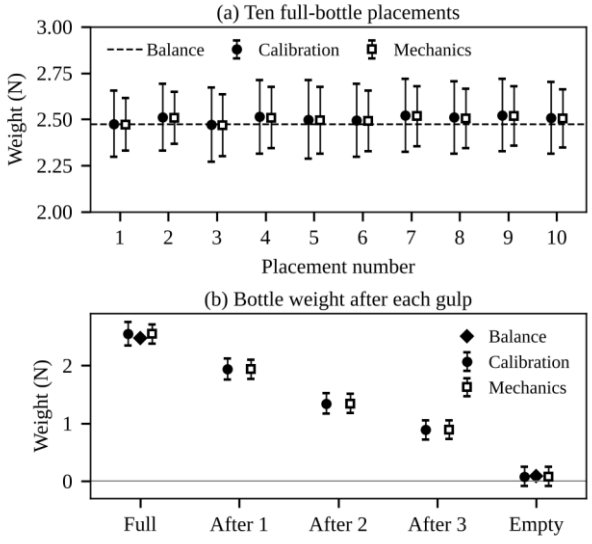


Fig. 6. Weights with propagated uncertainty bars from the calibration and mechanics methods. (a) Ten placements of the full bottle; every interval contains the balance reference. (b) The bottle after each gulp, with balance readings for the full and empty bottle; the weights fall from the full to the empty balance reading.

IV. Discussion

Both estimates agreed with the balance to about 1% for the full bottle, but their uncertainties are set mainly by the tared voltage and the calibration slope.

The calibrated mean exceeded the balance reference by 0.03 ± 0.16 N (1.1%), and the mechanics mean by 0.02 ± 0.11 N (0.9%). Neither deviation shows that one method is more accurate. Both use the same tared voltage, so their agreement tests only the slope, and the high R^2 supports only linearity over the tested loads. Both mean uncertainties include a 0.11 N tare term. Calibration adds a 0.11 N Monte Carlo slope term, whereas mechanics adds bounded geometry, modulus, gain, and gauge-factor terms totaling 0.03 N, so its smaller interval rests on those assumed bounds.

Beam theory with the Table I inputs predicts a slope magnitude of 9.75 N/V, within 0.1% of the fitted 9.76 N/V, so the calibrated and mechanics weights agree. The 2.085 setting is therefore consistent with the measured slope, although that alone cannot confirm the physical gauge factor. The fitted intercept, 0.04 ± 0.12 N, also agrees with the 0.054 N expected from the stored tare and the slope.

The control value 2.085 comes from the course set-up document [3] and reproduces the saved strain and weight outputs. The gauge packaging lists 2.0 ± 1%, so as a sensitivity check, a physical gauge factor of 2.0 would raise the mechanics weights by 4.25% and move the four known-weight checks to about +3.8%, +5.2%, +5.2%, and +3.9%. The fitted slope favors 2.085, but the data cannot statistically exclude 2.0, which implies a slope near 10.2 N/V against the fitted 9.76 ± 0.45 N/V. The entered modulus of 69.0 GPa was also retained; the supplier's typical 68.3 GPa differs by about 1%, against a course-rule modulus uncertainty of 0.14% [11].

Placement variation was small next to the shared calibration and tare terms. The ±0.013 N repeatability interval contributed about 0.6% of the calibrated mean's variance, and a single weighing would carry a Student-t interval of ±0.040 N (1.6%). All loads were centered on the same mark, and the sensitivity to offset is about 0.40% per millimeter. Position was not measured separately, and the placement scatter already contains replacement variability, so no extra position term was added.

In the gulp sequence the full-bottle reading was 2.55 ± 0.16 N by mechanics and 2.55 ± 0.20 N by calibration; both intervals contain the balance reference. Slope, loaded voltage, and tare contributed about 34%, 36%, and 30% of the calibrated variance. For mechanics, the tared voltage dominated (0.16 N), followed by the gauge factor (0.026 N) and gain (0.013 N); modulus and geometry each contributed under 0.005 N.

Observed scatter is roughly 6 to 10 times smaller than the stated intervals. The placement standard deviation was 0.017 N against stated uncertainties of 0.11 to 0.16 N, and the calibrated empty-bottle reading agreed with the balance to within 0.01 N against a stated 0.16 N. The unloaded baselines in Table III span only 3.6 mV (0.035 N), about a third of the 11 mV tare proxy. The intervals are wide because the course 2s convention describes the spread of individual samples, which is larger than the standard error of a block mean, and because the tare proxy is conservative. No instability of that size was observed.

Unloaded blocks averaged 5.6 to 8.3 mV against the 5.52 mV stored tare, offsets worth up to 0.027 N, comparable with the calibrated mean deviation. Using the placement file's own unloaded mean as the zero, with the slope fixed, changes its result to 2.53 N, or 2.1% above the balance, so re-zeroing shifts the bias without necessarily improving agreement. A stable gain error cancels in the calibrated weight because the fitted slope absorbs it, whereas the 0.5% gain bound adds 0.013 N to the mechanics weight; drift and nonlinearity were not quantified.

The full-bottle readings sit above the balance in every file: +1.1% for the placement mean, +2.1% when the placement file's own unloaded mean is used as the zero, +3.0% for the full bottle in the gulp file, and +3.2% for total consumption. The common sign suggests a systematic cause. Baseline drift (Table III), the slope, and a bottle center of gravity that differs from the brass weights are candidates; at 0.40% per millimeter, an offset of 5 to 8 mm would give 2% to 3%. The data do not separate these causes.

The largest and smallest gulps differed by 0.35 ± 0.29 N after including their shared reading. Total consumed weight was 2.47 ± 0.20 N, compared with the balance difference of 2.3936 ± 0.00071 N: a central excess of 3.2%. This comparison cancels the fixed tare. The empty-bottle uncertainty (0.16 N) was twice its weight (0.08 N), so a near-zero weight could not be told from zero. The gulp sizes were not distinguishable from one another, and their 0.15 N sample standard deviation is consistent with the measurement uncertainty. The full-bottle reading also differed between files (2.50 against 2.55 N), about 2%, which points to drift or placement.

Equation (7) estimates first yield at the clamp using clamp distance L_0 and yield strength σ_y. With the course-designated alloy [3], [4], a supplier's typical 276 MPa yield strength [11], and the measured 273.46 mm distance, the calculated load is 42.018 ± 0.070 N. The uncertainty reflects only the dimensional and course-rule bounds; the typical strength has no supplied batch tolerance, so this is a first-yield estimate and should not be read as a safe load. At that load the small-deflection model also fails: the tip deflection would be about 64 mm, 23% of L_0, whereas at 6.30 N it is about 9.5 mm (3.5%). The selected ±0.64 V amplifier window corresponds to an extrapolated load of 6.30 ± 0.29 N (slope term only), 60% beyond the tested maximum of 3.93 N and well below predicted yield. Large deflection and electronics can therefore limit use before material yielding.

$$W_y = \frac{\sigma_y b h^2}{6 L_0} \tag{7}$$

Only four reference loads, from 1.05 to 3.93 N, were used, and the largest was stacked from two weights. The fit therefore says nothing about linearity near the 6.30 N window limit. The tare statistics are a proxy because the original tare block was not saved. The amplifier window was found by trial and error, and no temperature or long-term drift test was run. Evaporation, drips, and the effect of resealing the cap were not measured, and the balance gave one reading per state, so they cannot be quantified here. The + − − + residual pattern of the calibration fit is what slight curvature or an error in a stacked label total would give, but four points cannot tell these apart. Repeating the lab with every load weighed on the balance, several saved tare blocks before each file, a placement jig for the bottle, and one load near the window limit should reduce the largest terms, tared voltage and slope, and would test linearity directly.

V. Conclusion

With the course-specified 2.085 gauge-factor setting, the two mean estimates were both 2.50 N, with uncertainties of 0.16 N calibrated and 0.11 N mechanics, and both contained the balance reference. The mechanics weights of the four reference loads agreed with their labels within 0.9%. Calibration showed no accuracy improvement over mechanics. Electrical scatter and tare uncertainty limited individual weighing, especially the empty bottle. The scale as built resolves loads of about 1 N and larger to roughly ±0.2 N, but not the near-empty bottle. The first-yield estimate of 42.018 ± 0.070 N lies far above the 6.30 ± 0.29 N amplifier-window limit that bounds use. The 2.0 package gauge factor (a sensitivity case) and material variability remain limits on absolute accuracy.

Appendix

A. Given and instrument uncertainties

Instrument bounds follow the instructor's guidance of 0.0001 in for the micrometer and about ten times that for the calipers, which is 0.00254 and 0.0254 mm before rounding [10]. No accuracy specification for the NEIKO caliper model is claimed. All bounds are treated as approximate 95% coverage, the common course basis, though a tolerance bound alone does not establish a probability distribution. The multimeter bound is ±(0.9% + 2 counts) of the reading [9]. Inputs and uncertainty symbols are listed together in Table VI.

TABLE VI
INPUTS AND ASSIGNED UNCERTAINTY BOUNDS

Quantity	Value	Uncertainty	Source / basis
Thickness	3.1370 mm	0.0025 mm	Instructor [10]
Width	25.3830 mm	0.0025 mm	Instructor [10]
Gauge-to-load distance	251.390 mm	0.025 mm	Instructor [10]
Clamp-to-load distance	273.460 mm	0.025 mm	Instructor [10]
Modulus	69.00 GPa	0.10 GPa	Course rule [8]
Gain	900 V/V	4.5 V/V	Course, 0.5% [8]
Control gauge factor	2.085	0.021	Assumed 1% [8]
Yield strength	276 MPa	0.1 MPa	Typical [11]; rule [8]
Reference weights	See Table VII	0.00050 N	Scale spec [8]

The 0.1 GPa and 0.1 MPa material bounds come from the course rule of 10% of the least reported order of magnitude [8], so they do not represent measured material scatter. The ±0.021 gauge-factor bound applies a 1% tolerance [8] to the 2.085 control setting as an assumption, and the 2.0 package case is treated separately as a sensitivity check. The gain bound (0.5% of 900) and the scale bound (±0.0005 N) are also course-specified [8]. Every reported interval depends on these bounds.

Each weight carries ±0.0005 N, so the stacked 3.9286 N load carries ±0.00071 N by RSS, while the single-weight loads carry ±0.0005 N. These are label values from the course scale, and no new balance measurements were made. Treating the weight errors as shared, independent, or absent changed the simulated slope uncertainty by less than 0.001 N/V, because the weight bounds are far smaller than the voltage terms. Table VII repeats the nominal inputs and coordinate uncertainties so the simulation can be reproduced.

TABLE VII
CALIBRATION COORDINATES AND ASSUMED UNCERTAINTIES

Mean amplifier voltage (V)	*Voltage uncertainty (V)*	*Reference weight (N)*	*Weight uncertainty (N)*
−0.1016	0.0087	1.0479	0.00050
−0.1979	0.012	1.9643	0.00050
−0.3055	0.012	3.0051	0.00050
−0.3962	0.011	3.9286	0.00071

B. Statistical uncertainties

The course uses twice the standard deviation for large sample sets and Student-*t* intervals for small sets [7, pp. 27–37]. Each voltage block contained 2,500 samples (Table VIII). Equation (8) defines placement sample variance; $\overline{W}$ is the mean of the N_p weights and s_p is their sample standard deviation. Equation (9) defines repeatability uncertainty U_{rep}; t is 2.262 for ten placements. The corresponding mechanics statistic uses W_m.

$$s_p^2 = \frac{\Sigma_i (W_{c,i} - \overline{W})^2}{N_p - 1} \tag{8}$$

$$U_{rep} = \frac{t\, s_p}{\sqrt{N_p}} \tag{9}$$

TABLE VIII
STATISTICAL INPUTS AND UNCERTAINTIES

Quantity	Value / count	Uncertainty / method
Excitation voltage	2.4991 V / 2,500	0.00030 V; twice the largest s.d.
Loaded voltage	Tables II, IV, V / 2,500	Twice the largest s.d. per block
Tare; U_0	0.0055 V / 2,500*	0.011 V proxy
Placement mean	10 per method	0.013 N; Student *t*

The tare proxy is twice the largest standard deviation among 18 unloaded blocks, 0.005605 V. The asterisk in Table VIII marks this proxy. Its 2,500 is the sample count of that block, used because the original tare statistics are unavailable. For the representative full-bottle block, V_a was −0.255760 V, s_{Va} was 0.006102 V, and V_s was 2.499116 V.

C. RSS propagation of (3) and (5)

$U_{\Delta V}$ is tared-voltage uncertainty; U_0 is tare uncertainty; U_C is slope uncertainty; U_{Wc} is calibrated-weight uncertainty. With independent loaded and tare noise, (10) and (11) give the required RSS forms. For independent inputs X, r_X denotes uncertainty divided by magnitude. The relative strain uncertainty is given by (12).

$$U_{\Delta V}^2 = U_{Va}^2 + U_0^2 \tag{10}$$

$$U_{Wc}^2 = (\Delta V_a\, U_C)^2 + (C\, U_{\Delta V})^2 \tag{11}$$

$$(U_\varepsilon/\varepsilon)^2 = r_{\Delta V}^2 + r_A^2 + r_{Gf}^2 + r_{Vs}^2 \tag{12}$$

D. RSS propagation of (4)

Equation (13) gives relative mechanics uncertainty r_m. It combines the relative uncertainties of modulus, width, load distance, amplifier gain, gauge factor, excitation voltage, and tared voltage with thickness counted twice, because weight depends on the square of thickness. Gain, gauge factor, and modulus enter as single relative terms; the tared-voltage term dominates a full-bottle reading because it does not shrink with the signal. U_{Wm} equals the magnitude of W_m times r_m.

$$\begin{aligned} r_m^2 = {} & r_E^2 + r_b^2 + 4r_h^2 + r_L^2 \\ & + r_A^2 + r_{Gf}^2 + r_{Vs}^2 + r_{\Delta V}^2 \end{aligned} \tag{13}$$

E. RSS propagation of (14)–(16)

Equation (14) subtracts the later state from the earlier state to define gulp G_i, with ΔV_i the corresponding earlier-minus-later voltage difference. The common tare cancels. Equation (15) retains independent block noise and the shared slope; $U_{V,i}$ denotes twice the saved standard deviation of state i.

$$G_i = W_{c,i-1} - W_{c,i} = C(V_{a,i-1} - V_{a,i}) \tag{14}$$

$$U_{Gi}^2 = (\Delta V_i\, U_C)^2 + C^2(U_{V,i-1}^2 + U_{V,i}^2) \tag{15}$$

For any linear combination of bottle voltages, define D in (16), with dimensionless coefficients a_i. This retains repeated-state covariance automatically. Total consumed weight Q uses coefficients (1, 0, 0, 0, −1); largest-minus-smallest gulp uses (0, 0, 1, −2, 1). The mean of these four observed gulps is one quarter of Q, so its propagated uncertainty is one quarter of UQ.

$$\begin{aligned} D &= C\, \Sigma_i\, a_i\, V_{a,i} \\ U_D^2 &= (\Sigma_i\, a_i\, V_{a,i}\, U_C)^2 + C^2\, \Sigma_i\, a_i^2\, U_{V,i}^2 \end{aligned} \tag{16}$$

F. RSS propagation of (17) and (18)

Common slope and tare uncertainty do not decrease with ten placements. Equation (17) combines them with U_{rep}; bars indicate means. Equation (18) gives the mechanics counterpart: H is the magnitude of weight sensitivity to voltage, and r_c includes all relative terms in (13) except $r_{\Delta V}$. The mechanics repeatability term uses its own sample standard deviation.

$$U_{mean,c}^2 = (\overline{\Delta V_a}\, U_C)^2 + (C\, U_0)^2 + U_{rep}^2 \tag{17}$$

$$\begin{aligned} H &= \frac{4Ebh^2}{6L\, A\, G_f\, V_s} \\ U_{mean,m}^2 &= (\overline{W_m}\, r_c)^2 + (H\, U_0)^2 + U_{rep}^2 \end{aligned} \tag{18}$$

G. RSS propagation of (7)

Yield-load uncertainty U_{Wy} follows (19), which combines the relative uncertainties of yield strength, width, doubled thickness, and clamp distance. Thickness is doubled because yield load depends on its square. Table VI gives each bound.

$$(U_{Wy}/W_y)^2 = r_{\sigma y}^2 + r_b^2 + 4r_h^2 + r_{L0}^2 \quad (19)$$

H. Monte Carlo calibration propagation

Following [2], each trial perturbed the four voltage coordinates normally, using the largest within-block standard deviations (half of each 95% interval), and the weights with standard deviation 0.00025 N each (half of the 95% bound); the 1.9643 N weight was shared by the 1.9643 N and 3.9286 N loads. Least squares fitted W against V_a in each trial, using C and B in (20); bars denote coordinate means.

$$C = \frac{\Sigma_i(V_{a,i} - \overline{V_a})(W_i - \overline{W})}{\Sigma_i(V_{a,i} - \overline{V_a})^2}, \qquad B = \overline{W} - C\,\overline{V_a} \quad (20)$$

The simulation used 120,000 trials. Twice the slope standard deviation gave $U_C = 0.45$ N/V, and the intercept bound was 0.12 N. Fig. 7 shows an approximately normal slope distribution and a convergence curve: 2σC settled near 0.446 N/V, varying by about 1% between 5,000 and 120,000 trials, so the trial count was sufficient. The mean simulated slope, −9.755 N/V, differed from the fitted −9.762 N/V by 0.07%, so perturbing the inputs did not bias the calibration. As a check of the implementation, the same procedure applied to the four-point and five-point examples in the course workbook [2] gave slope standard deviations of 0.097 and 0.054, against 0.097 and 0.053 stored in the workbook.

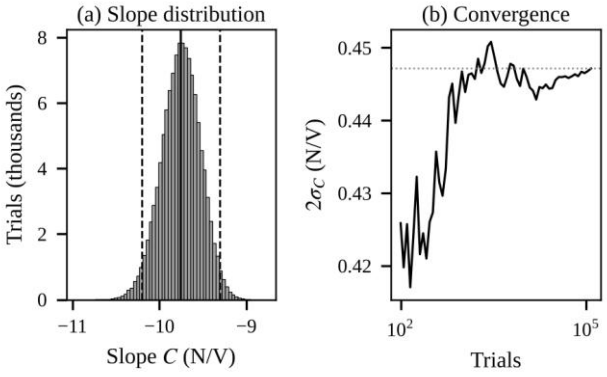


Fig. 7. Monte Carlo calibration slope. (a) Distribution of fitted slopes C for 120,000 trials; the solid line is the mean and dashed lines are the mean ± 2σ. (b) Twice the slope standard deviation versus trial count; the flat curve shows the 0.45 N/V uncertainty does not depend on the trial count.

I. Examples and propagated summary

Example calculation: the full-bottle block gave $\Delta V_a = -0.26128$ V and $U_{\Delta V} = 0.0166$ V; Equations (3) and (12) then gave $\varepsilon = -2.229 \times 10^{-4} \pm 1.4 \times 10^{-5}$. Equations (11) and (13) gave 2.55 ± 0.20 N calibrated and 2.55 ± 0.16 N mechanics. Equation (17), with mean ΔV_a of −0.25636 V, U_C of 0.45 N/V, U_0 of 0.011 V, and U_{rep} of 0.013 N, gave 0.16 N for the calibrated mean. The first-gulp voltage difference of −0.0629 V gave 0.61 ± 0.17 N. The remaining values were calculated the same way.

For capacity, (7) gives 42.018 N before rounding; (19) gives 0.070 N using the unrounded 0.00254 mm and 0.0254 mm bounds. The consumed-weight and gulp-difference bounds include shared states through (16). Propagated quantities are summarized with their dependence on the observation (Table IX).

TABLE IX
SUMMARY OF PROPAGATED UNCERTAINTIES

Quantity	Fixed / varies	Representative uncertainty
Tared voltage	Varies; (10)	0.017 V
Slope / intercept	Fixed; simulation	0.45 N/V / 0.12 N
Strain	Varies; (12)	0.000014
Individual calibrated weight	Varies; (11)	0.16–0.21 N
Individual mechanics weight	Varies; (13)	0.14–0.18 N
Gulp	Varies; (15)	0.17 N
Consumed weight	Fixed sequence; (16)	0.20 N
Gulp difference	Fixed contrast; (16)	0.29 N
Observed mean gulp	Fixed sequence	0.051 N
Mean calibrated / mechanics	Fixed dataset; (17), (18)	0.16 / 0.11 N
Yield load	Fixed model; (19)	0.070 N

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